

Linear Quantile Mixed-Effects Models

Matteo Bottai

Joint work with Nicola Orsini



**Karolinska
Institutet**

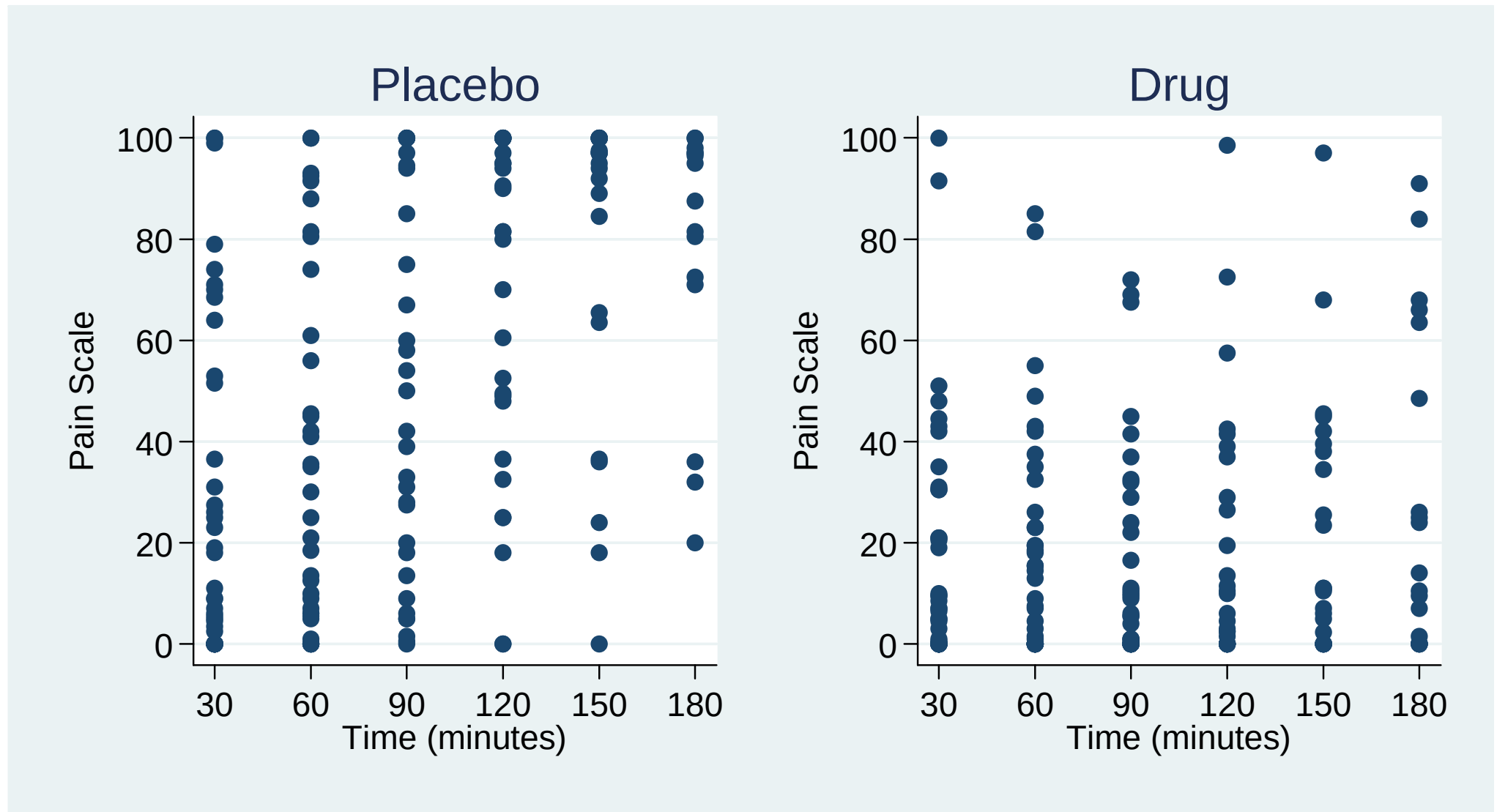


Longitudinal Data on Labor Pain

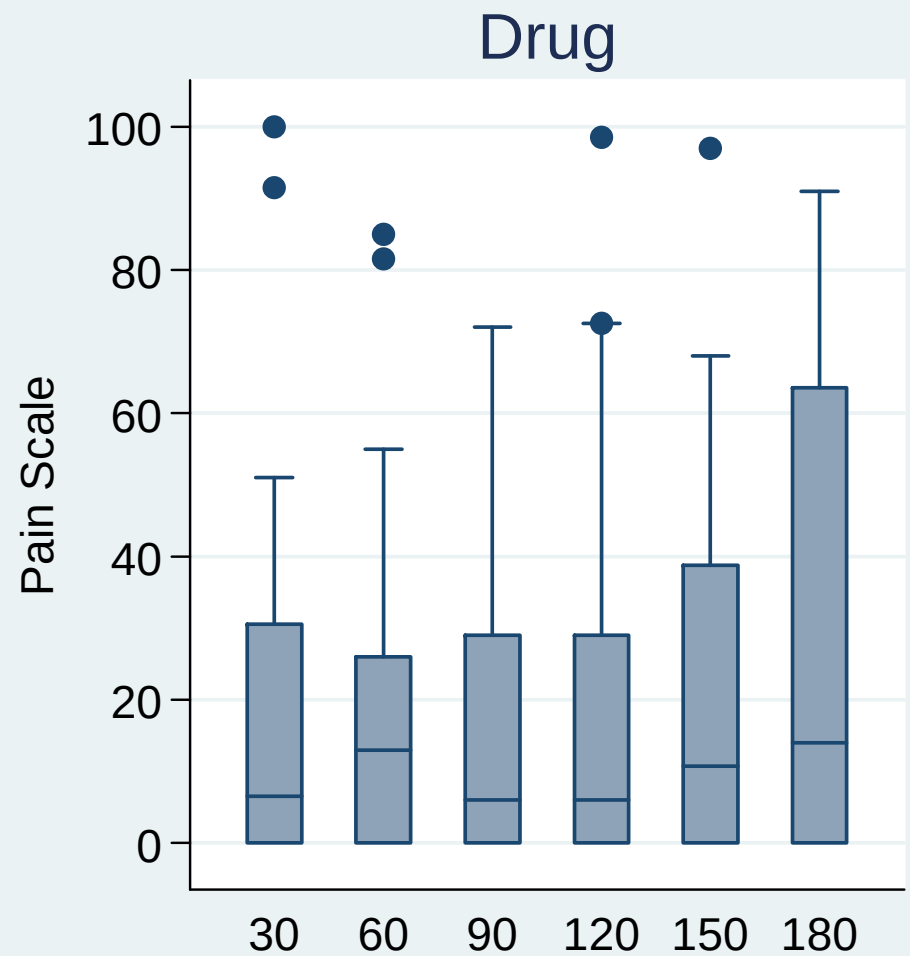
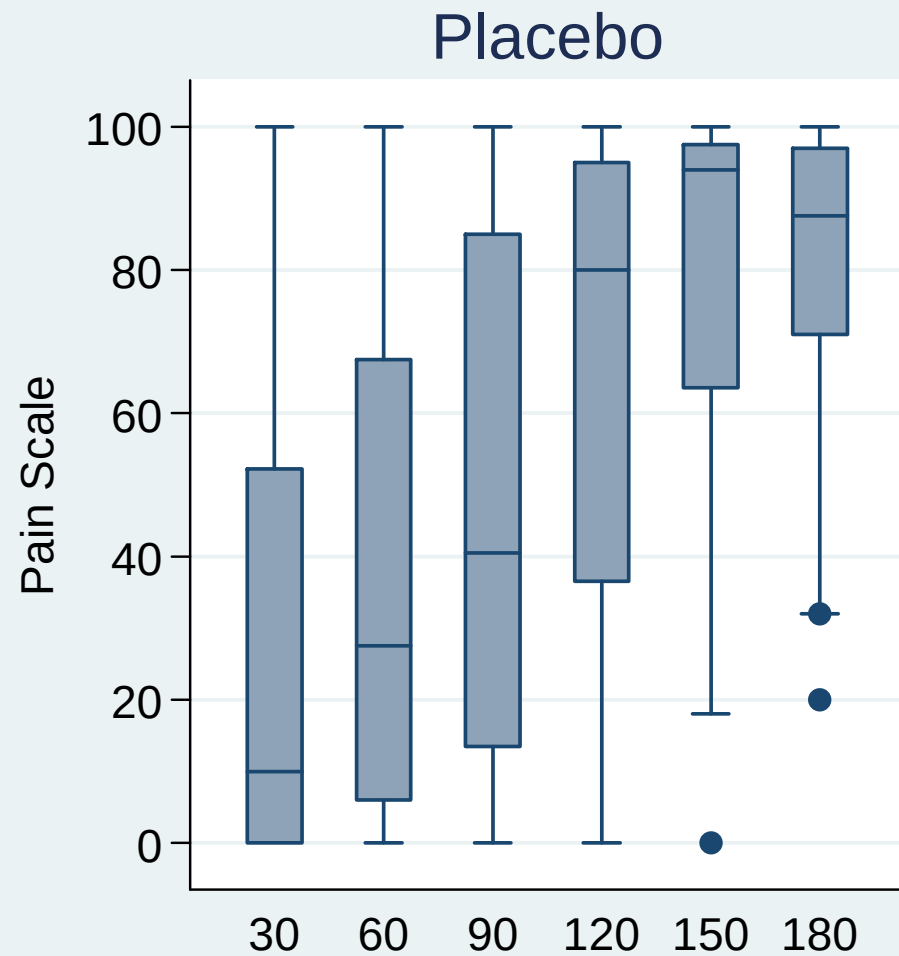
83 women in labor were randomized to drug or placebo (Davis, *Stat Med* 1991). They self-reported pain every 30 minutes on a 100-mm visual analogue scale.

treatment	subject	time	pain
1	1	30	0
1	1	60	0
1	1	90	0
1	2	30	0
1	2	60	0
1	2	90	0
1	2	120	2.5
1	2	150	2.3
1	2	180	14

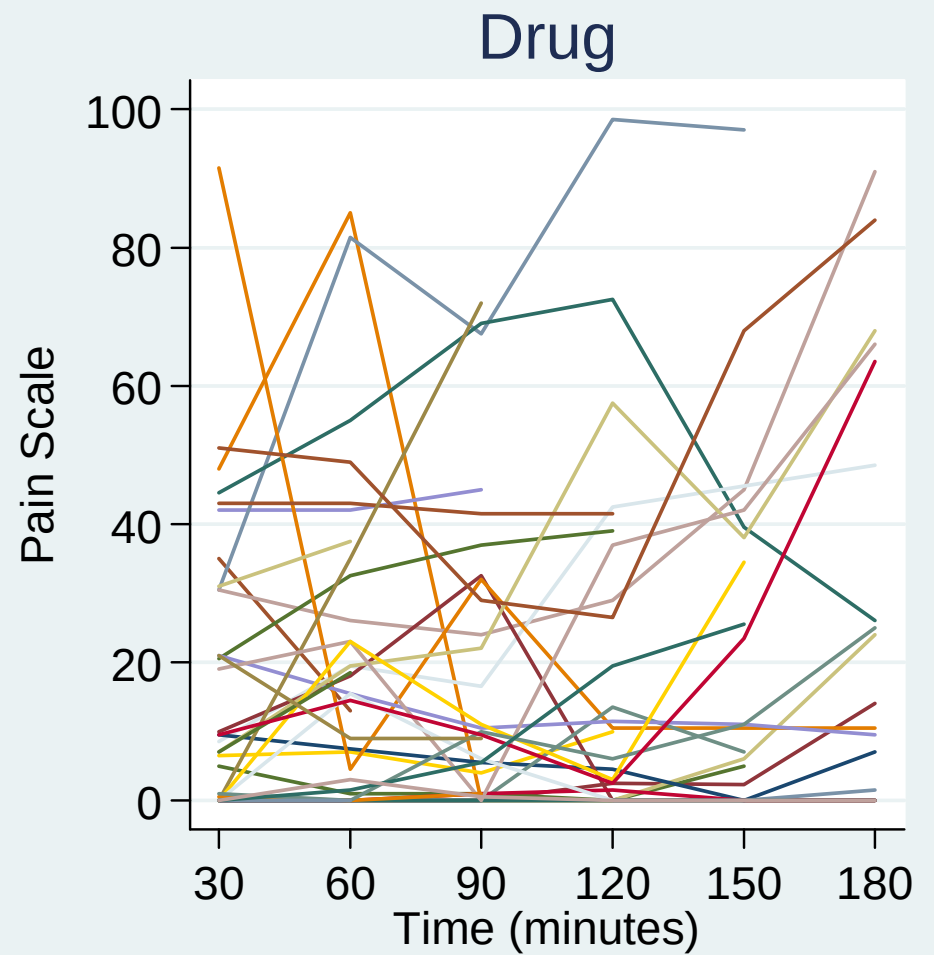
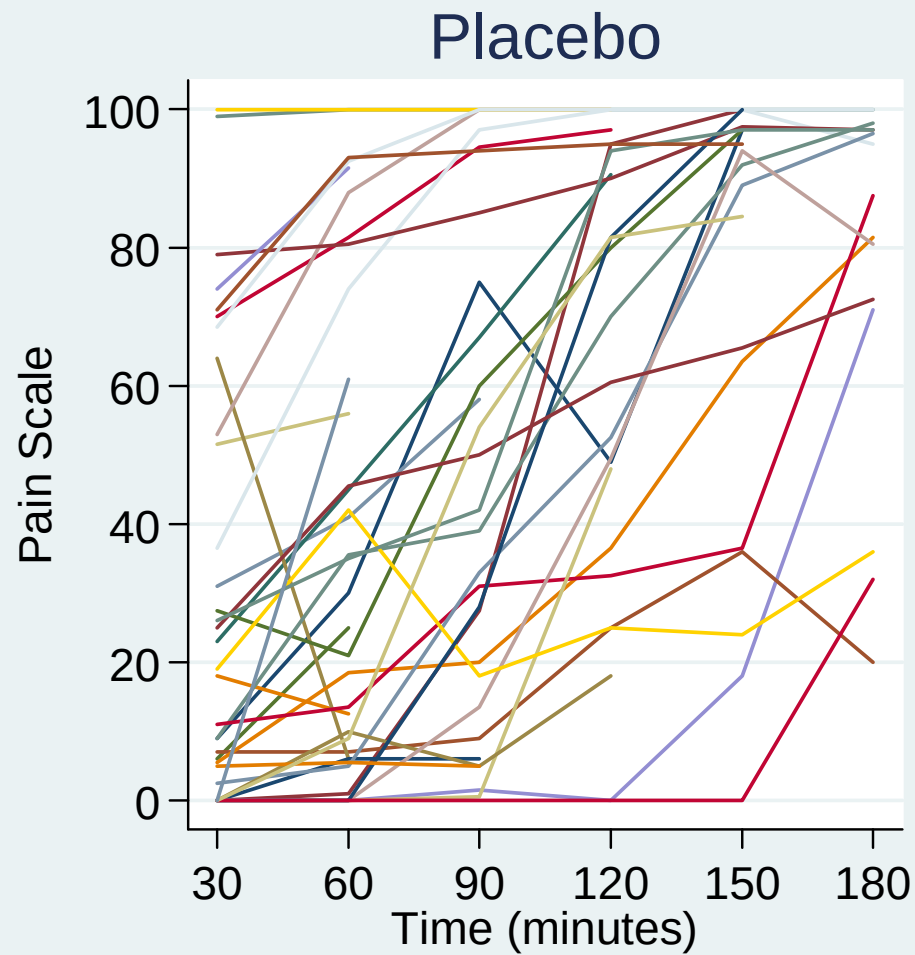
Pain over Time: a Scatter Plot



Pain over Time: a Box Plot



Pain over Time: a Spaghetti Plot



Logistic Quantile Mixed Effects Model

We consider a logistic quantile random-intercept regression model

$$\text{logit}(\text{pain}_{ij}) = \beta_0 + u_i + \beta_1 \text{time}_{ij} + \beta_2 \text{trt}_{ij} + \beta_2 \text{time}_{ij} \times \text{trt}_{ij} + \lambda \varepsilon_{ij}$$

for the j th observation on the i th woman.

We assume

$$u_i \sim N(0, \sigma^2)$$
$$\varepsilon_{ij} \sim AL(p): f(\varepsilon | p) = p(1 - p) \exp(I_{\varepsilon \leq 0} \varepsilon - p\varepsilon)$$

The conditional p -quantile of pain given covariates and random effects is

$$Q_{\text{pain}|\text{time},\text{trt},u}(p) = \text{logit}^{-1}(\beta_0 + u + \beta_1 \text{time} + \beta_2 \text{trt} + \beta_2 \text{time} \times \text{trt})$$

The “logit” link constrains inference on the p -quantile within 0 and 1.

A Prototype for the “xtqreg” Command

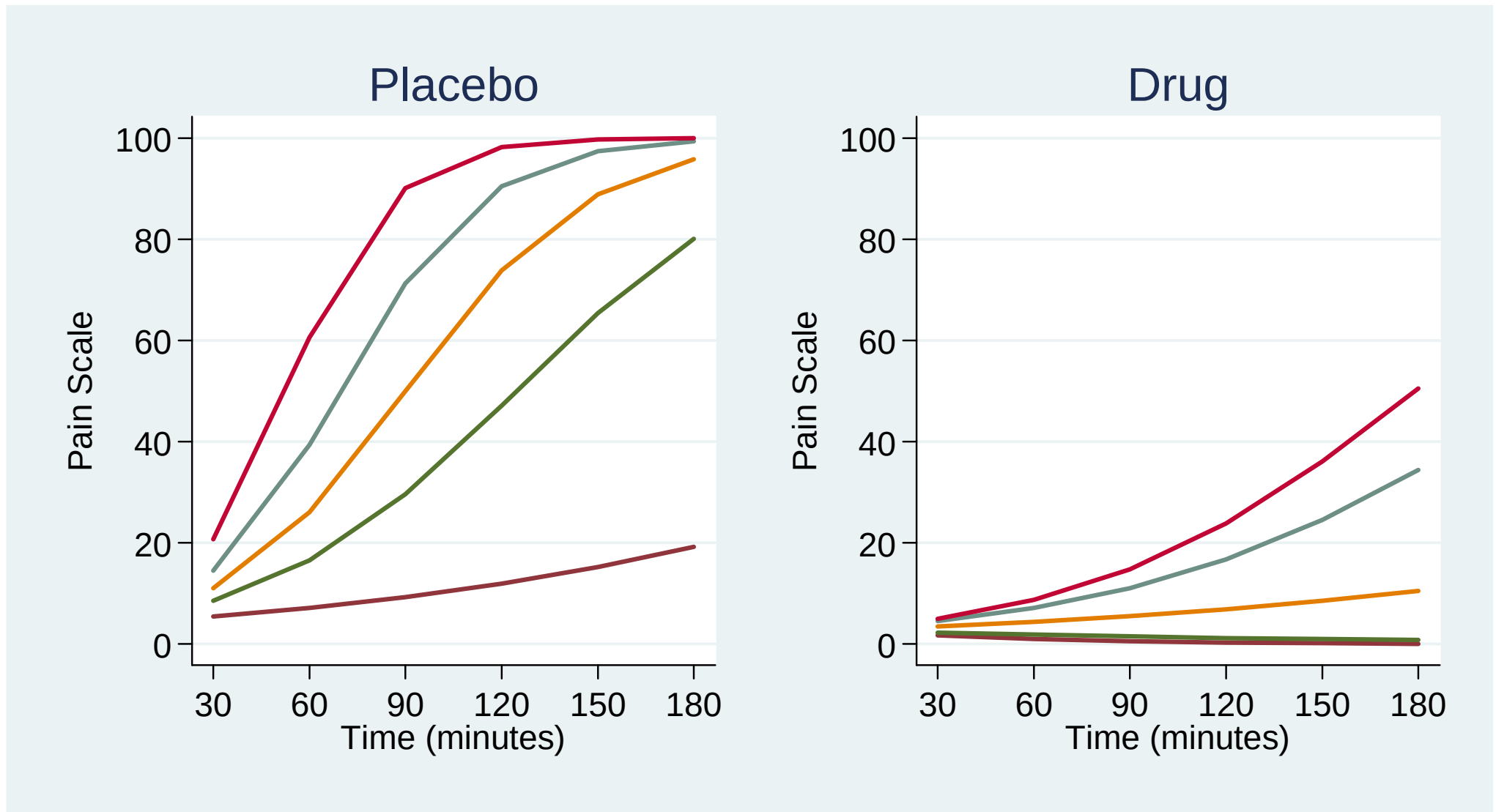
The prototype calls the R package “lqmm” developed by Marco Geraci.

```
. xtqreg logit_pain time treatment treatment_time, quantile(.1 .25 .5 .75 .9)
```

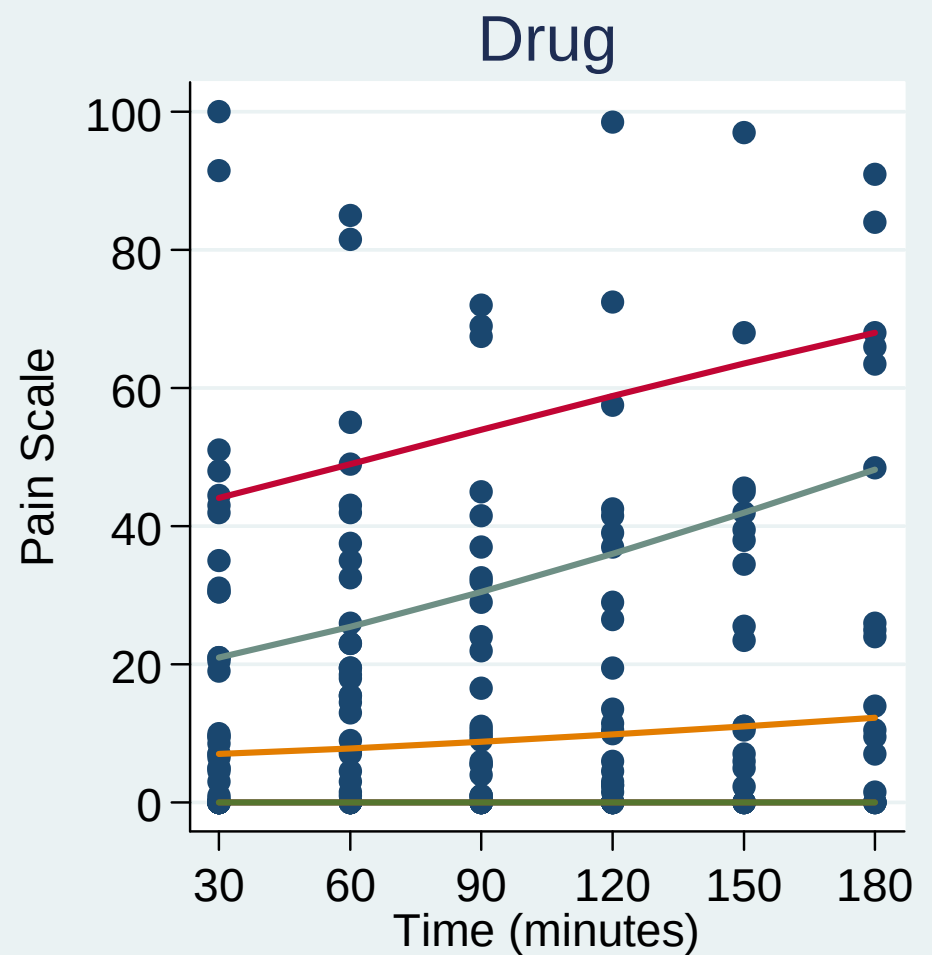
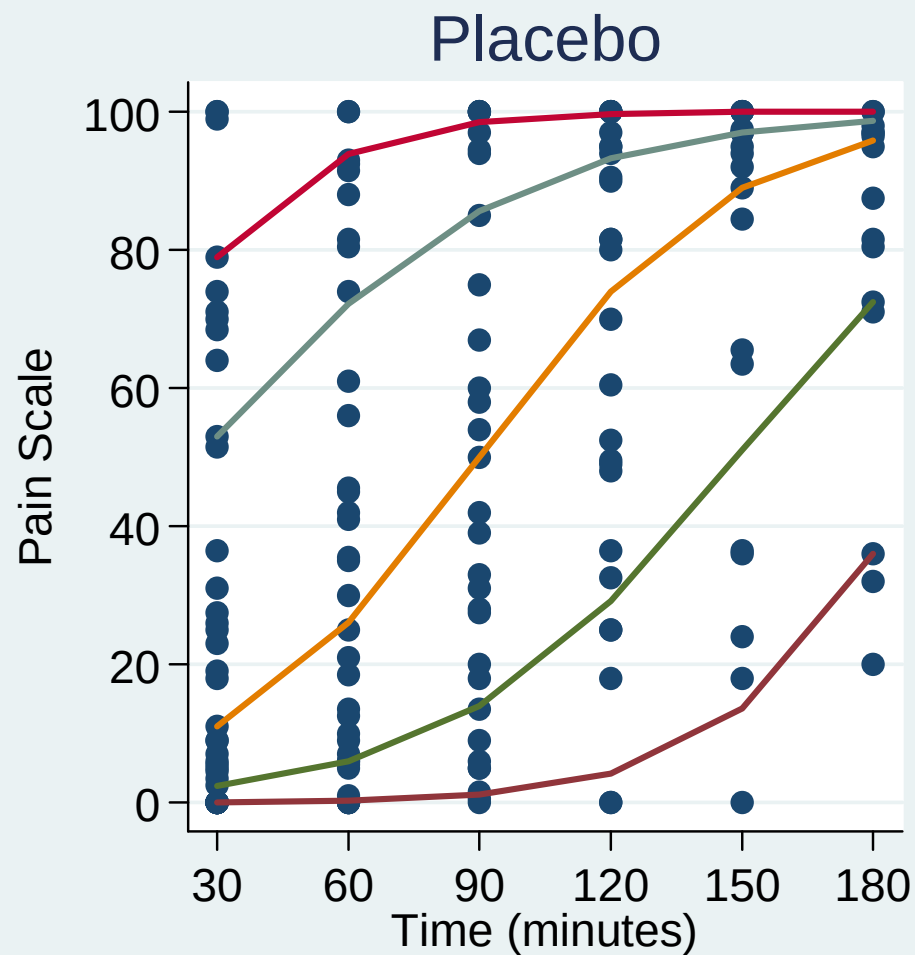
Mixed-effects Quantile regression	No. of obs	=	358
Optimization: Gradient-Search	No. of groups	=	83
Group variable: subject	No. of bootstrap	=	20
Covariance random-effects: Independent			

logit_pain		Coef.	Bootstrap Std. Err.	z	P> z	[95% Conf. Interval]	
[... output omitted ...]							
q50	time	.0345962	.0043228	8.00	0.000	.0261237	.0430687
	treatment	-.4051901	.8642464	-0.47	0.639	-2.099082	1.288702
	treatment_time	-.0268845	.005356	-5.02	0.000	-.037382	-.016387
	_cons	-3.115767	.60243	-5.17	0.000	-4.296509	-1.935026
[... output omitted ...]							

Conditional 10th, 25th, 50th, 75th, and 90th Percentiles



Marginal 10th, 25th, 50th, 75th, and 90th Percentiles



Possible Applications

Quantile mixed effects models can be applied to

- ✓ Longitudinal data
- ✓ Matched case-control data
- ✓ Twins cohort data
- ✓ Clustered data
- ✓ Spatially dependent data
- ✓ Multivariate outcomes
- ✓ Latent variable analysis

To do-list

- ✗ Implement the xtqreg command in Mata
- ✗ Improve computational stability and speed

xtqreg: Syntax

```
xtqreg depvar [indepvars] [if] [in] [, xtqreg_options]
```

xtqreg_options	Description

Model	
quantiles(numlist)	specifies quantiles; default is quantiles(.5)
reps(#)	generates # bootstrap samples; default is reps(20)
method(string)	specifies the optimization algorithm
random(varlist)	specifies the random effects
covariance(vartype)	specifies the variance-covariance of the random effects
noconstant	suppresses constant term
Reporting	
level(#)	set confidence level; default is level(95)

xtqreg: Random Effects Covariance Structures

vartype	Description
independent	one variance parameter per random effect, all covariances zero; the default unless a factor variable is specified
exchangeable	equal variances for random effects, and one common pairwise covariance
identity	equal variances for random effects, all covariances zero; the default for factor variables
unstructured	all variances and covariances distinctly estimated

See [R] sqreg postestimation for features available after estimation.

References

Linear Quantile Mixed Effects Models

Geraci and Bottai. Quantile regression for longitudinal data using the asymmetric Laplace distribution. *Biostatistics* 2007

Liu and Bottai. Mixed-Effects Models for Conditional Quantiles with Longitudinal Data. *International Journal of Biostatistics* 2009

Geraci and Bottai. Linear quantile mixed models. *Statistics and Computing* 2013

Logistic quantile regression

Bottai, Cai, and McKeown. Logistic quantile regression for bounded outcomes. *Statistics in Medicine* 2010

Orsini and Bottai. Logistic quantile regression in Stata. *Stata Journal* 2011